Supplemental Appendix

to "The Economics of Comparability of Treatment in Sovereign Debt Restructurings"

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This appendix collects extensions and additional results that are not needed to evaluate the findings of the paper, and the parameter values behind its figures. Sections, statements, equations, and figures numbered without the prefix "S" are those of the paper.

Two regimes recur. Under *commitment* the allocation is chosen at date 0, as in Assumption 5, and an optimum is a *Ramsey* allocation, as in Proposition 3. Under *discretion*, as in Corollary 2, it is chosen at date 1 in the restructuring state, taking q_P as predetermined: a *Markov* allocation, marked by the superscript M, maximizes the debtor's continuation utility at that node, and a Markov equilibrium requires in addition that investors anticipate it, $h_P^e = h_P^M$ in (2).

S.1 Additional Results for Section 3

S.1.1 When does Criterion 2 and 3 coincide?

The allocation of Criterion 3 is defined by the crisis-lending technology, whereas the no-windfall family $\{h_O^{\mathrm{nw}}(\kappa) : \kappa \in [0,1]\}$ of Criterion 2 is computable from contracts and prices. The first result characterizes when a netting share κ implements the Criterion 3's allocation.

Proposition S.1 (Implementation)**.** *Maintain the hypotheses of Proposition 2, and say that a netting share* $\kappa \in [0,1]$ implements *a target* h_O *if* $h_O^{\mathrm{nw}}(\kappa) = h_O$.

(i) (One-sidedness.) *The attainable set is* $\{h_O^{\mathrm{nw}}(\kappa) : \kappa \in [0,1]\} = \left[h_O^{\mathrm{nw}}(1),\, \theta\right]$, *with* $h_O^{\mathrm{nw}}(1) = 0$ *if and only if* $\kappa^* \equiv \mathcal{R}/(F\mathcal{R} + g_O d_P) \leq 1$. *No target above* θ *is implementable; in particular, full breach, the allocation for* $\pi < \hat{\pi}$ *(Proposition 3(ii)), is not.*

(ii) (Implementation.) *Let* $\pi > \hat{\pi}$, *so that Criterion 3 is* $h_O^\circ = h_O^\circ(\pi)$ *(Proposition 3(ii)), suppose* $h_O^\circ \leq \theta$, *and write* $h_P^\circ \equiv (\mathcal{R} - h_O^\circ d_O)/d_P$ *for the private haircut it requires.*

Criterion 3 is implementable if and only if

$$g_O \;\geq\; (1-F)\,h_P^\circ - h_O^\circ, \qquad \textit{in which case} \qquad \kappa^\circ \;\equiv\; \frac{h_P^\circ - h_O^\circ}{F h_P^\circ + g_O} \;\in [0,1]$$

implements it, and no other share does if $h_O^\circ > 0$. At the PCS allocation, $h_O^\circ = 0$, the share is $\kappa^\circ = \kappa^*$, every $\kappa \in [\kappa^*, 1]$ implements it, and the condition reads $\kappa^* \leq 1$; it fails at $g_O = 0$, where $\kappa^* = 1/F > 1$.

Proof in Appendix S.4.1. The rule can protect the official class relative to equal treatment but never burden it more: the compensation differential it nets, $F h_P^{\mathrm{nw}} + g_O$, is positive. Below θ, the share must recognize exactly enough of that differential to cover the treatment differential Criterion 3 demands, $h_P^\circ - h_O^\circ$. The rule's reach is thus bounded by what was priced at origination, whereas the Criterion 3 protects the official class for emergency-lending capacity, which is never priced (Assumption 2(ii)) and is invisible to any compensation-based measure. Where κ° implements Criterion 3, the allocations coincide but the measured justification (realized concessions) differs from the welfare one (prospective access); and unpriced protection can be justified only up to the priced concessions that exist, which is why $g_O = 0$ forecloses full exemption. At the illustrative parameters of Table S.1, full netting lowers the official haircut only from $\theta = 43.9$ to 37.1 percent, against 6.9 percent under Criterion 3 at $\pi = 0.5$: no netting share implements it.

S.1.2 Robustness to Assumption 2(ii), (iii)

The second result shows what Assumption 2(ii) (officials don't price risk) contributes to Criteria 2 and 3.

Corollary S.1 (Risk-reflective official pricing)**.** *Replace Assumption 2(ii) by official terms that price the anticipated haircut like the market: per unit of face value the official lender disburses $q_O = \beta^*(1 - F h_O^e) + s$ at date 0, where h_O^e is the anticipated official haircut, equal to h_O in equilibrium, and $s \geq 0$ is a fixed subsidy.*

(a) (Criterion 3.) *Date-0 consumption, $c_0 = y_0 + \beta^* D - \beta^* F \mathcal{R} + s\, d_O$, is invariant to the allocation. Hence, if $\underline{h} < h_{\mathrm{a}}$, the Ramsey allocation is preferred creditor status for every π and every s, and it coincides with the Markov allocation of Corollary 2.*

(b) (Criterion 2.) *Maintain the hypotheses of Proposition 2. Then $\chi_O = F h_O^e - R^* s$, and for every $\kappa \in [0,1]$ the no-windfall rule has a unique equilibrium, in which*

$$h_P - h_O \;=\; \min\left\{\tau, \frac{\mathcal{R}}{d_P}\right\}, \qquad \tau \equiv \frac{\kappa R^* s}{1 - \kappa F}.$$

At $s = 0$ it is equal treatment, $h_P = h_O = \theta$, for every κ.

Proof in Appendix S.4.2. Unpriced official losses at the margin drive both the commitment problem and part of the no-windfall treatment differential. Were they priced, shifting relief between classes would move the two issue prices in exactly offsetting ways and only access would matter, so full protection would be both optimal and credible: what creates the incentive to promise official burden-sharing is the margin, not concessionality on average (the subsidy s). Under Criterion 2, Assumption 2(ii) is what places the premium Fh_P^e in χ_P with no counterpart in χ_O, so that a differential survives at $g_O = 0$ (Proposition 2). With symmetric pricing this risk-compensation component unwinds, and only a pure grant sustains differential treatment.

The third result shows what Assumption 2(iii) contributes to Criterion 3, by letting private investors lend in the crisis.

Proposition S.2 (Backup finance)**.** *Maintain Assumptions 1–4, except that in the date-2 liquidity-crisis state the government may also borrow any amount $b \geq 0$ from private investors at R^*, repaid at settlement, and suppose $\inf_c u'(c) < 1$. Let $\Delta_\ell^b(h_O) \equiv \min\{\Delta_\ell(h_O),\, 1 - \beta R_E\}$.*

(i) At every $h_O < h_a$ it is optimal to draw the facility in full, uniquely so if $R_E < R^*$, and the marginal value of its capacity is $\Delta_\ell^b(h_O)$; private borrowing is positive if and only if $u'(c_2^c + \bar{\ell}(h_O)) > 1$, in which case $\Delta_\ell^b(h_O) = 1 - \beta R_E$.

(ii) If $R_E < R^*$, Lemma 1, Proposition 3, and Corollary 2 hold with Δ_ℓ^b in place of Δ_ℓ in (15), (16), and the thresholds; in particular $\pi_p^b = u'(c_0(\underline{h}))/[\beta\lambda\Delta_\ell^b(\underline{h})] \geq \pi_p$, and the discretionary allocation is still preferred creditor status.

(iii) If $R_E = R^*$, the Ramsey allocation is full breach, $\overline{h}$, for every π, and the date-1 planner is indifferent among all feasible allocations.

Proof in Appendix S.4.3. The facility is drawn first because it is cheaper, and the private loan tops consumption up to the point where $u' = 1$. Where the top-up is active, official capacity is worth only what it saves in interest (the grant element); where it is not, cheap official funds have pushed crisis consumption past that point and the wedge, though positive, is below the grant element. Either way the scarcity premium of (11) is gone: private borrowing supplies the liquidity it priced. Preferred creditor status then rests on the rate subsidy alone. At the parameters of Table S.1, the grant element is 0.019 of the wedge 0.353 at $h_O = 0$, the protection threshold rises from $\pi_p = 0.64$ to $\pi_p^b = 11.8$, and full breach is optimal at every $\pi \in (0, 1)$: the protection in Figure 2 is bought by liquidity that the market does not supply, not by the two-point subsidy.

S.1.3 Robustness to the normalization of excess payoffs

Remark S.1 (Levels per unit of face value, not rates of return). x_i is an excess payoff per unit of face value: the difference between what the claim pays and the capitalized price paid for it, both per unit of promised face. It is not a rate of return on the funds advanced; the corresponding rate is $x_i/(R^*q_i)$, and because origination prices differ across classes ($q_P = \beta^*(1 - Fh_P^e) < \beta^* \leq 1/R_O = q_O$ when $h_P^e > 0$), equalizing per-face payoffs is not equalizing rates: were payoffs equalized, the official class (having paid more per unit of face) would still bear the smaller percentage loss on funds advanced. Three considerations justify the face-value normalization. The first relates to units: haircuts (Definition 1) and the relief constraint (6) are per-unit-of-face-value, date-1 present-value objects, so Criteria 1 and 2 equalize measurements in the same units and differ only by the netting of χ_i (the adjustment is the criterion's entire content). Second, the practice the criterion formalizes compares treatment per claim: CoT assessments (Section 5) are stated as NPV losses per unit of contractual claim. Third, the rate-equalizing analogue, which is the grant-element proposal of Section 5.3, replaces the linear condition (14) with a relation nonlinear in the anticipated haircut (through $q_P(h_P^e)$); Proposition S.3 below solves it, with a quadratic in place of the linear closed forms of Proposition 2, a lower exemption cutoff, and a larger official haircut, the direction the first sentence of this remark implies. The equilibrium discipline of Lemma 2(ii) is normalization-free: since q_i is predetermined, expected excess payoffs remain zero for P and nonpositive for O in rates as well. Only the cross-class equalization of realized outcomes depends on the choice of units, which is why we call the criterion origination-adjusted treatment rather than equal returns on investment.

The last result of this section solves the rate-normalized rule of Remark S.1, which Section 5.3 identifies with the grant-element procedure, in the form of Proposition 2.

Proposition S.3 (Netting per unit of funds advanced)**.** *Maintain Assumption 2(ii) and* $\mathcal{R} \leq d_P$*, as in Proposition 2, and let the* rate-normalized rule *minimize* $\left|(h_P - \chi_P)/(R^*q_P) - (h_O - \chi_O)/(R^*q_O)\right|$*, the gap between the post-relief grant elements* $1 - (1 - h_i)/(R^*q_i)$ *of Section 5.3, subject to* (6) *and* $h_i \in [0, 1]$*, given the measured* (χ_i, q_i) *of Lemma 2, with equilibrium as in Definition 4. The rule has a unique equilibrium. It is PCS,* $h_O = 0$ *and* $h_P = \mathcal{R}/d_P$*, if and only if* $\mathcal{R} \leq g_O d_P/(1 - F + g_O)$*; otherwise* $h_O \in (0, \overline{h})$*, with*

$$h_P = \frac{2C}{B + \sqrt{B^2 - 4Fd_PC}}, \qquad h_O = \frac{(1 - F + g_O)h_P - g_O}{1 - Fh_P}, \qquad B \equiv d_P + (1 - F + g_O)d_O + F\mathcal{R}, \quad C \equiv \mathcal{R} + g_O d_O.$$

Relative to full face-normalized netting ($\kappa = 1$ *in Proposition 2), the official class is exempt on a weakly smaller set of relief requirements,* $g_O d_P/(1 - F + g_O) \leq g_O d_P/(1 - F)$*, and bears a weakly larger haircut, strictly if and only if* $\mathcal{R} > g_O d_P/(1 - F + g_O)$*.*

Proof in Appendix S.4.4. The rate-normalized rule measures each class's netted loss against its own outlay, and the official class advanced the most per unit of promised face, $q_O = 1/R_O \geq \beta^* \geq q_P$. Scaled by that outlay, its concession counts for less, $g_O/(1+g_O)$ rather than g_O, and so does each unit of haircut imposed on it, $1/(1+g_O)$ against $1/(1-Fh_P) > 1$ for a unit imposed on the private class, whose outlay the spread reduced. The rule therefore recognizes a smaller official concession and needs a larger official haircut to close a given gap: it brings the official class in at a lower relief requirement and, once in, assigns it more relief than face-value netting, through the private base alone when $g_O = 0$. The self-reference of Definition 4 is intact, since the private base $R^* q_P = 1 - Fh_P$ is the market's price of the anticipated treatment; it is what makes the equilibrium the root of a quadratic rather than linear in $\mathcal{R}$.

S.2 Model Extension with Three Creditor Classes

The paper's official lender is a multilateral crisis lender. This section adds bilateral official creditors, who lend for development goals or geopolitical reasons, and sometimes at concessional terms. This development lending is sometimes conditioned on the treatment received on their claims. These represent a third class of claims and a second source of future flows. One parameter, the intensity $\gamma \geq 0$ with which those flows are conditioned on the treatment of bilateral claims, nests the traditional Paris Club creditor ($\gamma = 0$) and the lender that conditions new lending on treatment ($\gamma > 0$). We maintain $\mathcal{R} \leq d_P$, as in Proposition 2: private claims can absorb the whole relief requirement, so protecting both official classes is feasible.

S.2.1 Environment

Add to the economy of Section 2 a bilateral official class B: face value $d_B > 0$ due at date 1, issued at date 0 at a contractual gross rate $R_B \leq R^*$ that does not respond to the anticipated allocation.[1] As in Section 2.2, $g_B \equiv R^*/R_B - 1 \geq 0$ is its concessionality margin. The grant element of the loan at the risk-free rate is $\mathrm{GE}_B \equiv 1 - R_B/R^* = g_B/(1+g_B)$. The date-0 budget becomes $c_0 = y_0 + q_P d_P + d_O/R_O + d_B/R_B$. Assumption 3 and Definition 1 extend to the three claims with $D \equiv d_P + d_O + d_B$, so the relief constraint is $h_P d_P + h_O d_O + h_B d_B = \mathcal{R}$ with $h_i \in [0,1]$; in relief units $r_i \equiv h_i d_i$, it reads $r_P + r_O + r_B = \mathcal{R}$ with $r_i \in [0, d_i]$.

[1] Bilateral lenders price risk coarsely: the concessionality that qualifies a loan as official development assistance is set by recipient income group (OECD, 2014), export-credit premia are minimum rates by country-risk category (OECD, 2024), and Chinese policy banks manage repayment risk through collateralized revenue accounts, cross-default provisions, and "no Paris Club" clauses rather than through repricing (Horn, Reinhart, and Trebesch, 2021; Gelpern et al., 2023). None reprices the anticipated allocation at the margin; coarse risk compensation acts like the subsidy s of Corollary S.1(a), a level shift that opens no pricing channel. What responds to treatment is future lending (export-credit cover, the clauses above), the margin that γ captures in Assumption S.1.

Assumption S.1 (Bilateral development window)**.** On the restructuring branch, the bilateral creditor operates a development window at date 2: in *each* date-2 state the government may borrow $\ell_B \in [0, \bar{\ell}_B(h_B)]$ at the gross rate R_B, repaying $R_B \ell_B$ at settlement, with capacity

$$\bar{\ell}_B(h_B) \;=\; \max\{0,\; \bar{\ell}_{B0} - \gamma\, h_B d_B\}, \qquad \gamma \geq 0, \quad \bar{\ell}_{B0} \geq 0, \tag{S.1}$$

and capacity $\bar{\ell}_{B0}$ on the repayment branch. Scarcity extends to the window: $u'(y^h - \bar{M} + \bar{\ell}_{B0}) > \beta R_B$ and $u'(y^\ell - \bar{M} + \bar{\ell}_0 + \bar{\ell}_{B0}) > \beta \max\{R_E, R_B\}$.

Acyclical window. We distinguish lending for development goals from crisis lending noting the former disburse flows on program and project schedules. Thus, the window operates in both date-2 states. The evidence finds aid disbursements volatile and, if anything, mildly procyclical (Pallage and Robe, 2001; Bulíř and Hamann, 2008), so state independence is a conservative middle ground. *Conditioning intensity.* The parameter γ is the future capacity withdrawn per unit of bilateral relief; linearity, as in (8), keeps the planner's problem concave while both windows retain capacity. Like λ, it is an own-access response (Section 2.4), here of policy and contractual rather than balance-sheet origin: about zero for the traditional Paris Club lender, which granted relief repeatedly and tolerated arrears, and positive for the lender whose contracts put new flows at stake (footnote 1 above). The window is *inert* if $\gamma \bar{\ell}_{B0} = 0$: it puts no future flows at stake. *A single rate.* The window lends at the rate R_B of the inherited claims: donor policy sets terms by income category, not loan by loan, so one rate is the minimal representation. *Scarcity.* As in Assumption 4, date-2 liquidity stays scarce with both facilities unimpaired, so both caps bind. As $\beta R_B \leq 1$, the conditions on R_B hold whenever marginal utility at the two consumption levels of the assumption exceeds its settlement value, $u' > 1$.

Definition S.1 (Window surpluses and wedges)**.** *For capacities $(\bar{x}_O, \bar{x}_B)$ with $0 \leq \bar{x}_O \leq \bar{\ell}_0$ and $0 \leq \bar{x}_B \leq \bar{\ell}_{B0}$, the surplus in state y^h is*

$$A^h_B(\bar{x}_B) \;\equiv\; \max_{0 \leq \ell_B \leq \bar{x}_B} \; u(y^h - \bar{M} + \ell_B) - \beta R_B \ell_B \;-\; u(y^h - \bar{M}), \tag{S.2}$$

and the joint surplus in state y^ℓ, which reduces to the access surplus (9) at $\bar{x}_B = 0$, is

$$A^\ell(\bar{x}_O, \bar{x}_B) \;\equiv\; \max_{\substack{0 \leq \ell_O \leq \bar{x}_O \\ 0 \leq \ell_B \leq \bar{x}_B}} \; u(c^c_2 + \ell_O + \ell_B) - \beta R_E \ell_O - \beta R_B \ell_B \;-\; u(c^c_2). \tag{S.3}$$

The wedges are

$$\Delta^h_B(\bar{x}_B) \equiv u'(y^h - \bar{M} + \bar{x}_B) - \beta R_B, \qquad \Delta^\ell_B(\bar{x}_O, \bar{x}_B) \equiv u'(c^c_2 + \bar{x}_O + \bar{x}_B) - \beta R_B,$$
$$\Delta_\ell(\bar{x}_O, \bar{x}_B) \equiv u'(c^c_2 + \bar{x}_O + \bar{x}_B) - \beta R_E,$$

the last of which equals the wedge $\Delta_\ell(h_O)$ *of* (10) *at* $(\bar{x}_O, \bar{x}_B) = (\bar{\ell}(h_O), 0)$*, and* $\bar{\Delta}_B \equiv \pi\Delta_B^\ell + (1-\pi)\Delta_B^h$ *is the expected bilateral wedge.*

Under Assumption S.1, both caps bind at every such pair (Appendix S.4.6), so $A^\ell(\bar{x}_O, \bar{x}_B) = u(c_2^c + \bar{x}_O + \bar{x}_B) - \beta R_E \bar{x}_O - \beta R_B \bar{x}_B - u(c_2^c)$ and the three wedges are strictly positive. Then A^ℓ is jointly concave in $(\bar{x}_O, \bar{x}_B)$ and submodular, $\partial^2 A^\ell / \partial \bar{x}_O \partial \bar{x}_B = u'' \leq 0$, so the two official facilities are substitutes in state y^ℓ: each unit of one lowers the marginal value of the other.

Remark S.2 (Nesting). With $d_B = 0$ and $\bar{\ell}_{B0} = 0$ the economy is that of the paper. With $d_B > 0$ it is not, even if the window is inert: an unconditioned bilateral claim is still a third margin from which relief can be sourced.

Fix $\mathcal{R}$. The accounting of Lemma 1 extends to three classes (Appendix S.4.5): in any competitive equilibrium, date-0 expected utility is, up to an allocation-independent constant,

$$\mathcal{W}(h_O, h_B) \;=\; u(c_0(h_O, h_B)) + \beta^2 F\Big[\pi\, A^\ell\big(\bar{\ell}(h_O), \bar{\ell}_B(h_B)\big) + (1-\pi)\, A_B^h\big(\bar{\ell}_B(h_B)\big)\Big], \tag{S.4}$$

with $c_0(h_O, h_B) = y_0 + \beta^*(d_P - F\mathcal{R} + Fh_O d_O + Fh_B d_B) + d_O/R_O + d_B/R_B$, strictly increasing in both official haircuts. The allocation thus operates through three objects: the date-0 price, the emergency facility, which enters only the liquidity-crisis state, and the development window, which enters both date-2 states.

S.2.2 Marginal Costs of Relief with Three Classes

The marginal costs of relief (16) extend to three classes. With the wedges of Definition S.1 evaluated at the capacities $(\bar{\ell}(h_O), \bar{\ell}_B(h_B))$, let

$$\sigma_P \equiv \beta^* F\, u'(c_0), \qquad \sigma_O \equiv \beta^2 F \pi \lambda\, \Delta_\ell\, \mathbf{1}\{\bar{\ell}(h_O) > 0\}, \qquad \sigma_B \equiv \beta^2 F \gamma\, \bar{\Delta}_B\, \mathbf{1}\{\bar{\ell}_B(h_B) > 0\}$$

denote the cost, in the welfare function (S.4), of a unit of relief sourced from each class; $\sigma_B = 0$ at every allocation if and only if the window is inert.

Proposition S.4 (Three classes under discretion and commitment)**.** *Maintain* $\mathcal{R} \leq d_P$.

(a) (Discretion.) *Every Markov allocation has* $h_O^M = 0$*. If* $\gamma > 0$ *and* $\bar{\ell}_{B0} > 0$*, the Markov allocation is unique, with* $h_B^M = 0$ *and* $h_P^M = \mathcal{R}/d_P$*, and its issue price,* $q_P^M = \beta^*(1 - F\mathcal{R}/d_P)$*, is the lowest attainable under any feasible allocation. If the window is inert, every feasible allocation with* $h_O = 0$ *is a Markov allocation: the date-1 planner is indifferent over the split of relief between* P *and* B*.*

(b) (Commitment with an inert window.) *There is a Ramsey allocation that sources relief from* B *first,* $h_B = \min\{1, \mathcal{R}/d_B\}$*, and every Ramsey allocation with* $h_P > 0$ *does so. The*

pair (h_P, h_O) *then maximizes* $\mathcal{W}$ *subject to* $h_P d_P + h_O d_O = \max\{0, \mathcal{R} - d_B\}$*, a two-class problem to which Proposition 3 applies when* $\mathcal{R} > d_B$*.*

(c) (Commitment with conditioning.) *Let* $\gamma > 0$ *and* $\bar{\ell}_{B0} > 0$*. At any Ramsey allocation at which both windows retain capacity, there is a multiplier* $\mu > 0$ *such that* $\sigma_i = \mu$ *for every class with* $h_i \in (0,1)$*,* $\sigma_i \leq \mu$ *for every class with* $h_i = 1$*, and* $\sigma_i \geq \mu$ *for every class with* $h_i = 0$*. In particular, if* $h_P \in (0,1)$*, then* $h_B \in (0,1)$ *requires*

$$u'(c_0)\,\beta^* \;=\; \beta^2\gamma\big[\pi\,\Delta_B^{\ell} + (1-\pi)\,\Delta_B^{h}\big], \tag{S.5}$$

and $\beta\gamma\bar{\Delta}_B > u'(c_0)$ *at that allocation implies* $h_B = 0$*. If no feasible allocation exhausts a window,* $\lambda \min\{d_O, \mathcal{R}\} < \bar{\ell}_0$ *and* $\gamma \min\{d_B, \mathcal{R}\} < \bar{\ell}_{B0}$*, the Ramsey allocation is unique and these conditions characterize it.*

Proof in Appendix S.4.6. Recoveries are ordered by what a haircut destroys, with no assumption about legal seniority. Cutting the multilateral destroys crisis access; cutting bondholders destroys date-0 revenue through the price; cutting a bilateral lender whose future flows are neither priced at the margin nor conditioned on treatment destroys nothing the planner values. Such a lender is the residual claimant of the restructuring: co-residual with bondholders under discretion and weakly junior to both other classes at the allocation of part (b), $h_B \geq \max\{h_P, h_O\}$, where the multilateral–private ranking follows the regimes of Proposition 3 in the residual problem. Schlegl, Trebesch, and Wright (2019) document this ordering: multilateral claims are senior, and bilateral official claims are junior, or at least not senior, to private ones.

Preferred status is induced, but is not granted. Under discretion the price is sunk, so a class is protected only by a payoff-relevant state that its haircut would destroy: the multilateral has one (Corollary 2), and any $\gamma > 0$, however small, gives another to a bilateral lender whose window has capacity. Against a convention under which an unconditioned lender takes a haircut h_B, conditioning therefore lowers the issue price by $\beta^* F h_B d_B / d_P$: every unit of bilateral exposure it shields is a unit of anticipated bondholder loss. Under commitment, protection is earned at the margin: (S.5) is the access margin (15) with $\gamma\bar{\Delta}_B$ in place of $\pi\lambda\Delta_\ell$, and the bilateral class overtakes bondholders once $\beta\gamma\bar{\Delta}_B$ exceeds $u'(c_0)$. In each date-2 state the bilateral wedge is a scarcity premium plus the grant element, $u' - \beta R_B = (u' - 1) + \mathrm{GE}_B$, because $\beta R_B = 1 - \mathrm{GE}_B$: conditioned flows are worth at least their concessionality whenever date-2 marginal utility is at least its settlement value, $u' \geq 1$. Where both windows retain capacity, $\mathcal{W}$ is strictly concave, so the conditions of part (c) hold at one such allocation at most; allocations that exhaust a window are compared with it by value, as in Proposition 3(ii).

Remark S.3 (Comparative statics of the bilateral margin). Hold the multilateral haircut fixed

at some $h_O < h_a$, and let $h_B^\circ \in (0,1)$, with $\bar{\ell}_B(h_B^\circ) > 0$, solve (S.5). Then

$$\frac{\partial h_B^\circ}{\partial \gamma} < 0, \qquad \frac{\partial h_B^\circ}{\partial g_B} < 0, \qquad \operatorname{sign} \frac{\partial h_B^\circ}{\partial \pi} = \operatorname{sign} \left[\bar{\ell}(h_O) - (y^h - y^\ell)\right].$$

Proof in Appendix S.4.6. At a given multilateral haircut, official bilateral haircuts fall with conditioning intensity and with the concessionality of the flows at stake. Crisis risk has no monotone effect because the window is a subsidy, not insurance against crises: it lends in both date-2 states, so more crisis risk raises its value, and lowers the bilateral haircut, only if the emergency facility leaves the liquidity-crisis state the poorer one, $\bar{\ell}(h_O) < y^h - y^\ell$.

S.2.3 No-Windfall Equilibrium with Three Classes

Lemma 2 applies verbatim to the bilateral class: the one-period zero-coupon claim issued at R_B has pre-paid compensation $\chi_B = -g_B$. We extend the rule of Definition 4 to three classes: it returns the profile (h_P, h_O, h_B) with $h_i = \min\{1, \max\{0,\, n + \kappa\chi_i\}\}$ for every class, the level n chosen to meet the relief constraint. Classes off the bounds then bear the netted loss n, an exempt class ($h_i = 0$) at least n, and a class written off in full ($h_i = 1$) at most n; with two classes the profile is the allocation of Definition 4. As there, an equilibrium is a profile that the rule returns when investors anticipate it, $\chi_P = F h_P$.

Corollary S.2 (The no-windfall equilibrium with three classes)**.** *Add class B, with $D = d_P + d_O + d_B$, maintain the hypotheses of Proposition 2, and suppose $g_B \geq g_O$; the rule uses neither λ nor γ, so the reverse case follows by exchanging the labels O and B. For each $\kappa \in [0,1]$ the rule has a unique equilibrium. At an anticipated profile the three netted losses $h_i - \kappa\chi_i$ are equal if and only if $(1-\kappa F)h_P = h_O + \kappa g_O = h_B + \kappa g_B$, which together with $h_P d_P + h_O d_O + h_B d_B = \mathcal{R}$ has the unique solution*

$$h_P^{\mathrm{nw}}(\kappa) = \frac{\mathcal{R} + \kappa\,(g_O d_O + g_B d_B)}{d_P + (1-\kappa F)(d_O + d_B)}, \qquad h_i^{\mathrm{nw}}(\kappa) = (1-\kappa F)\, h_P^{\mathrm{nw}}(\kappa) - \kappa g_i, \quad i \in \{O, B\}. \tag{S.6}$$

The function $h_B^{\mathrm{nw}}(\cdot)$ is strictly decreasing, so either $h_B^{\mathrm{nw}} > 0$ on all of $[0,1]$ (in which case set $\kappa_3^* \equiv 1$) or there is a unique $\kappa_3^* \in (0,1]$ with $h_B^{\mathrm{nw}}(\kappa_3^*) = 0$. (i) For $\kappa \leq \kappa_3^*$ the equilibrium is the profile (S.6); on this range $h_P^{\mathrm{nw}} > \max\{h_O^{\mathrm{nw}}, h_B^{\mathrm{nw}}\}$ for $\kappa > 0$, $h_O^{\mathrm{nw}} - h_B^{\mathrm{nw}} = \kappa(g_B - g_O)$, and h_P^{nw} is strictly increasing in κ, from $h_P^{\mathrm{nw}} = h_O^{\mathrm{nw}} = h_B^{\mathrm{nw}} = \theta$ at $\kappa = 0$. (ii) For $\kappa > \kappa_3^*$ no feasible profile equalizes the netted losses, and in equilibrium class B is exempt, $h_B = 0$; with $\kappa^* \equiv \mathcal{R}/(F\mathcal{R} + g_O d_P) \geq \kappa_3^*$, the other two are $h_P = (\mathcal{R} + \kappa g_O d_O)/(d_P + (1-\kappa F)d_O)$ and $h_O = (1-\kappa F)h_P - \kappa g_O$ for $\kappa \leq \kappa^*$, as in Proposition 2(i), and $h_O = 0$, $h_P = \mathcal{R}/d_P$ for $\kappa \geq \kappa^*$.

Proof in Appendix S.4.7. The logic of Proposition 2 survives a second official class: the private class's anticipated haircut is priced at issuance and the premium is netted as pre-paid com-

pensation, so a given netted loss corresponds to a private haircut larger by the factor $1/(1-\kappa F)$ and to an official haircut smaller by the recognized concession, κg_i. The issue price falls as the netting share rises, until both official classes are exempt. What is new is the ranking within the official sector: Criterion 2 orders official creditors inversely by the concessionality of the claims they already delivered, so the more concessional class never takes the larger haircut and is exempt whenever the other is. At full netting this is the sequential participation of Section 5.3, stated per unit of face value; per unit of funds advanced the ordering within the official sector is the same, since at a common post-relief grant element $e < 1$ equalization sets $h_i = e - (1-e)g_i$, decreasing in g_i (Proposition S.3 for two classes). Nothing forward-looking enters the ranking, the conditioning intensity γ included.

Discussion. The window capacity $\bar{\ell}_{B0}$ is, like $\bar{\ell}_0$, a stand-in for a resource constraint the model does not derive. Neither own-access response is directly measured: contracts reveal whether new bilateral lending is conditioned on treatment, not by how much, and capital frameworks give the gearing of institution-wide headroom, not its pass-through to one borrower (Section 2.4).

S.3 Illustration Details

For the paper's two-class economy, Table S.1 lists the parameter values behind Figure 2. They are chosen so that the three regimes of Proposition 3 occur within the probability domain, not as a calibration, and no frequency is attached to them; F and π are the probabilities of a restructuring event and of a liquidity crisis, not re-default rates.

Table S.1: Illustrative parameter values ($u = \log$)

Parameter	Value	Interpretation	Note
R^*	1.04	gross world risk-free rate	4%
$\beta = \beta^*$	1/1.04	discounting (Assumption 1)	—
F	0.20	restructuring probability	illustrative
π	0.50	liquidity-crisis prob. post-restructuring	illustrative; horizontal axis of Figure 2
λ	3	own-access response, $\lambda = sL$ (Section 2.4)	illustrative; varied in Table S.2
R_E, R_O	1.02	concessional rates	grant element $1 - R_E/R^* = 1.9\%$
d_P, d_O	0.35, 0.25	predetermined composition	official share ≈42%
$\bar{M}$	0.35	sustainability envelope	implies $\theta = 43.9\%$
y_0, y^n, y^c, y^h, y^ℓ	1, 1, 0.8, 1, 0.6	endowments	deep date-2 crisis
$\bar{\ell}_0$	0.50	unimpaired emergency capacity	scarcity condition holds

Table S.2 varies the own-access response λ around its value in Table S.1, $\lambda = 3$, holding the other parameters fixed. For $\lambda \leq 2$, $h_a \geq \overline{h}$: the access-preserving branch is the whole feasible set, the lower threshold is π_a, and the allocation is continuous in π; for $\lambda \geq 3$, $h_a < \overline{h}$, the lower threshold is π_b, and the allocation jumps there (Proposition 3(ii)). At $\lambda \leq 1.5$ the protection

threshold π_p exceeds one, so the official class is never fully protected within the probability domain; at $\lambda \geq 4$ it is exempt already at $\pi = 0.5$.

Table S.2: Sensitivity of the regimes of Proposition 3 to the own-access response λ

λ	h_{a}	$\hat{\pi}$	π_p	h_O^*	h_P^*	q_P	$\bar{\ell}(h_O^*)$
1	2.00	0.65	1.93	1.000	0.038	0.954	0.25
1.5	1.33	0.26	1.29	0.574	0.343	0.896	0.28
2	1.00	0.11	0.96	0.291	0.545	0.857	0.35
3	0.67	0.25	0.64	0.069	0.703	0.826	0.45
4	0.50	0.27	0.48	0.000	0.753	0.817	0.50
6	0.33	0.27	0.32	0.000	0.753	0.817	0.50

Notes: Table S.1 parameters, $u = \log$. $\hat{\pi} = \pi_a$ for $\lambda \leq 2$ and π_b for $\lambda \geq 3$ (Proposition 3(ii)); full breach is optimal below $\hat{\pi}$ and preferred creditor status above $\max\{\hat{\pi}, \pi_p\}$. The last four columns are the Ramsey allocation at $\pi = 0.5$, the issue price it implies, and the emergency capacity it leaves. The baseline is $\lambda = 3$; under the backup-finance benchmark of Proposition S.2, $\pi_p = 11.8$ at that value.

Table S.3 reports the parameterization behind Figure 3. It is Table S.1 with two amendments made to fit Proposition 5. First, $\bar{\ell}_0$ rises from 0.50 to 0.65: coexistence requires the official relationship to retain value at the pessimistic allocation, and at $\bar{\ell}_0 = 0.50$ capacity at h_O^d falls to 0.016, below the official payment $m_O^d = 0.092$—violating the servicing bound (Appendix A.8), so the default assessment collapses into total default, where the metric's distortion is symmetric (Lemma 4(i)) and the taxonomy degenerates. Second, y^h rises from 1 to 1.3, widening the admissible cost range $[\bar{M}/y^h, \bar{M}/y^\ell)$ so that the emptiness regime of Proposition 5(v) clears the serviceability floor; at $y^h = 1$ it falls below the floor and is simply not visible. Neither change affects the baseline numbers, which remain at Table S.1. The new parameter is φ, the horizontal axis of Figure 3; $\varphi = 0.40$ is inside the coexistence window.

Table S.3: Parameter values for the exit-yield extension

Parameter	Value	Interpretation	Note
φ	0.40	default cost (share of y_2)	inside window $[0.340, 0.430)$
y^h	1.3	date-2 recovery endowment	Table S.1: 1; widens $[\bar{M}/y^h, \bar{M}/y^\ell)$
$\bar{\ell}_0$	0.65	unimpaired emergency capacity	Table S.1: 0.50; keeps $S(h_O^d) \geq 0$
all others	Table S.1	—	scarcity condition holds ($u'(0.90) > \beta R_E$)

Remark S.4 (Recovery and cost timing). With recovery rate $\rho \in [0, 1)$ on defaulted claims, all pricing formulas of Section 4 hold with π replaced by the loss intensity $\pi(1 - \rho)$, and the government's default gains shrink by the recovered amounts. The timing of the cost, by contrast, is substantive. If φy_2 were levied once per defaulted class rather than once per default event, total default would incur $2\varphi y^\ell$ while official-only default incurs φy^ℓ, so the ranking of these two actions in Lemma 3 reverses whenever $\varphi y^\ell > m_P$: defaulting on the official class alone (paying m_P and forgoing the emergency window) can then be optimal. The selective-default

characterization is therefore specific to the event-level cost, consistent with φ as the output disruption of default status.

Remark S.5 (Width of the coexistence window). In Proposition 5, the private-margin conditions of parts (i) and (ii) hold together iff $\varphi y^{\ell} \in [m_P(\theta), m_P^d)$, a window of width $R^* d_P(\theta - h_P^d)$ with $\theta - h_P^d = \pi d_O(D - \mathcal{R})/\big(D(D - \pi d_O)\big)$: it widens mechanically with post-restructuring crisis risk π. Whether the coexistence set as a whole inherits this monotonicity depends on how the official-service condition $S(h_O^d) \geq 0$ responds to the mandated official haircut, which rises with π ($dh_O^d/d\pi = (D - \mathcal{R})d_P/(D - \pi d_O)^2 > 0$), and the assumptions of Lemma 3 do not sign that response: with $b + m_O$ fixed at $(1 - \varphi)y^{\ell}$, a higher h_O raises the post-default base one-for-one with the fall in m_O while shrinking emergency capacity, so on the binding-cap branch with $\bar{\ell}(h_O) > 0$, $S'(h_O) = d_O[(R^* - \lambda)u'(b + \bar{\ell}) + \lambda\beta R_E]$ (positive whenever $\lambda \leq R^*$ and of either sign otherwise) while at an interior borrowing optimum $S'(h_O) = \beta R_E R^* d_O > 0$. We therefore leave the overall comparative static in π unsigned.

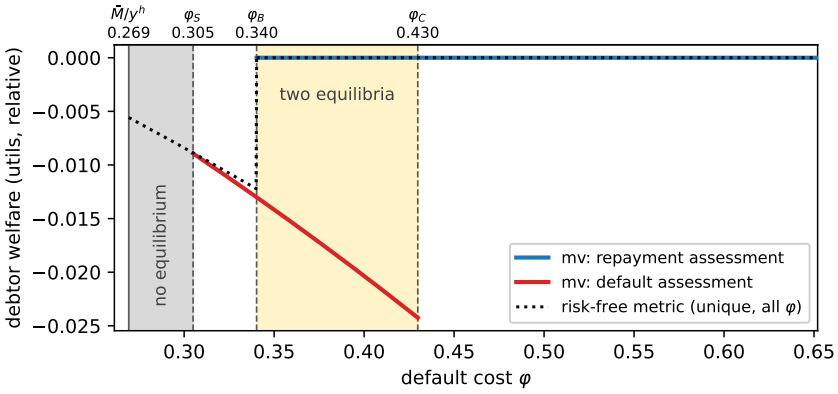


Figure S.1: Debtor welfare across assessments (Table S.3): welfare along each branch, by default cost φ (utils, relative to the repayment assessment). The dotted line is the risk-free metric, whose jump at $\varphi_B = m_P(\theta)/y^{\ell}$ marks the fundamental disappearance of date-2 default at θ. The risk-free line is defined everywhere and unique; the market-value correspondence is missing on the gray band and double-valued on the amber one, with the pessimistic branch strictly below.

Remark S.6 (Debtor welfare across assessments). Where the two assessments of Proposition 5 coexist, only the date-0 price term and the value of the restructuring branch in state y^{ℓ} differ across them (the decomposition of Lemma 1, with state y^h identical by Lemma 3). The debtor's date-0 welfare in the repayment and default assessments, $\mathcal{W}^{rep}$ and $\mathcal{W}^d$, therefore satisfies

$$\mathcal{W}^{rep} - \mathcal{W}^d = \Big[u\big(c_0^{rep}\big) - u\big(c_0^d\big)\Big] + \beta^2 F\pi\Big[\Gamma\big(\bar{\ell}(\theta); c_2^c\big) - \Gamma\big(\bar{\ell}(h_O^d); c_2^c + m_P^d - \varphi y^{\ell}\big)\Big],$$

where c_0^{rep} and c_0^d equal $y_0 + \beta^*(1 - F\eta)d_P + d_O/R_O$ at $\eta = \theta$ and at $\eta = h_O^d$ (Lemma 4(ii)).

The first bracket is strictly positive, because pessimism lowers the issue price. The second is ambiguous in general: the default assessment embeds a state-contingent transfer (the base gains $m_P^d - \varphi y^\ell > 0$ in the crisis-upon-crisis state), while the repayment assessment keeps more access ($\bar{\ell}(\theta) - \bar{\ell}(h_O^d) > 0$). At the parameters of Table S.3 the second bracket is also strictly positive throughout the coexistence window: at $\varphi = 0.40$ the debtor loses 0.020 utils (0.009 through the date-0 price, 0.011 through state y^ℓ), equivalent to 2.0 percent of date-0 consumption, and the gap ranges over $[0.013, 0.024]$ across the window (Figure S.1). Since private investors break even ex ante under either assessment and the official lender recovers less in the default one (Proposition 5(iv)), the pessimistic equilibrium at these parameters is a pure coordination failure adjudicated by a bookkeeping convention.

S.4 Proofs for Sections S.1 and S.2

S.4.1 Proof of Proposition S.1

(i) By Proposition 2(i), h_O^{nw} is continuous on $[0,1]$, strictly decreasing from $h_O^{\mathrm{nw}}(0) = \theta$ on $[0, \min\{\kappa^*, 1\}]$, and equal to zero on $[\kappa^*, 1]$ when $\kappa^* \leq 1$; its range is therefore $[h_O^{\mathrm{nw}}(1), \theta]$, with $h_O^{\mathrm{nw}}(1) = 0$ iff $\kappa^* \leq 1$. Full breach lies above this range: $\overline{h} = \min\{1, \mathcal{R}/d_O\} > \mathcal{R}/D = \theta$, since $\mathcal{R} < D$ and $d_O < D$.

(ii) Let $h_O^\circ \in (0, \theta]$. If $h_O^{\mathrm{nw}}(\kappa) = h_O^\circ$ for some $\kappa \in [0,1]$, then $\kappa < \kappa^*$, so the equilibrium solves (14) and (6): $h_P^{\mathrm{nw}}(\kappa) = h_P^\circ$ by (6), and (14), with $\chi_P = F h_P^\circ$ and $\chi_O = -g_O$, reads $h_P^\circ - h_O^\circ = \kappa(F h_P^\circ + g_O)$, that is, $\kappa = \kappa^\circ$. Hence κ° is the only candidate, and it is admissible iff $\kappa^\circ \leq 1$, that is, iff $g_O \geq (1-F)h_P^\circ - h_O^\circ$; note $\kappa^\circ \geq 0$ because $h_P^\circ - h_O^\circ = D(\theta - h_O^\circ)/d_P \geq 0$. Conversely, if $\kappa^\circ \leq 1$, the pair (h_P°, h_O°) solves (14) and (6) at $\kappa = \kappa^\circ$ with nonbinding bounds ($h_O^\circ > 0$ and $h_P^\circ < \mathcal{R}/d_P \leq 1$), so by the uniqueness in Proposition 2 it is the equilibrium. Now let $h_O^\circ = 0$, so that $h_P^\circ = \mathcal{R}/d_P$ and $\kappa^\circ = \mathcal{R}/(F\mathcal{R} + g_O d_P) = \kappa^*$. By Proposition 2(i), $h_O^{\mathrm{nw}}(\kappa) = 0$ exactly for $\kappa \in [\kappa^*, 1]$, a nonempty set iff $\kappa^* \leq 1$, that is, iff $g_O d_P \geq (1-F)\mathcal{R}$, which is the displayed condition at $h_O^\circ = 0$; and at $g_O = 0$, $\kappa^* = 1/F > 1$. □

S.4.2 Proof of Corollary S.1

(a) With rational expectations, $c_0 = y_0 + q_P d_P + q_O d_O = y_0 + \beta^* D - \beta^* F(h_P d_P + h_O d_O) + s\, d_O$, and the bracket equals $\mathcal{R}$ by (6). Nothing else in the proof of Lemma 1 involves the official price, so $\mathcal{W}(h_O)$ is a constant plus $\beta^2 F \pi A(\bar{\ell}(h_O))$, which is strictly decreasing on $[\underline{h}, h_{\mathrm{a}})$ and zero beyond (proof of Corollary 2). If $\underline{h} < h_{\mathrm{a}}$, its unique maximizer is $\underline{h}$, which is also the unique date-1 optimum of that proof.

(b) Since $\beta^* R^* = 1$, $\chi_O = 1 - R^* q_O = F h_O^e - R^* s$, and $\chi_P = F h_P^e$. With compensations

measured at an anticipated allocation (h_P, h_O), the netted gap of Definition 4, $(h'_P - \kappa\chi_P) - (h'_O - \kappa\chi_O)$, is strictly decreasing in h'_O along (6) and equals $(1-\kappa F)(h_P - h_O - \tau)$ at the anticipated allocation itself. The rule returns that allocation if and only if this value is zero, or nonpositive with $h_O = 0$, or nonnegative with $h_O = \overline{h}$. The first case requires $h_P - h_O = \tau$, feasible iff $\tau \leq \mathcal{R}/d_P$: by (6), $h_O = (\mathcal{R} - \tau d_P)/D$ and $h_P \leq \mathcal{R}/d_P \leq 1$. The second, where $h_P - h_O = \mathcal{R}/d_P$, requires $\tau \geq \mathcal{R}/d_P$. The third is impossible, since $h_P < \theta < \overline{h}$ there. Hence the equilibrium is unique, with $h_P - h_O = \min\{\tau, \mathcal{R}/d_P\}$; at $s = 0$, $\tau = 0$ and $h_P = h_O = \theta$. □

S.4.3 Proof of Proposition S.2

Using $\beta R^* = 1$, define $\Phi(c) \equiv \max_{b\geq 0}\{u(c+b) - b\}$ for $c > 0$. Since u' is continuous and strictly decreasing, with $\inf u' < 1$ and $u'(c) \to \infty$ as $c \to 0$, there is a unique $c^\dagger > 0$ with $u'(c^\dagger) = 1$; the maximizer is $b(c) = \max\{0, c^\dagger - c\}$, and $\Phi(c) = u(c^\dagger) - (c^\dagger - c)$ for $c \leq c^\dagger$ and $\Phi(c) = u(c)$ for $c \geq c^\dagger$, so Φ is concave and continuously differentiable with $\Phi'(c) = \min\{u'(c), 1\}$. In the crisis state the value of capacity x is $V(x) \equiv \max_{0\leq\ell\leq x}\{\Phi(c_2^c + \ell) - \beta R_E \ell\}$, the settlement of the private loan being netted in Φ as that of the facility is in (9).

(i) The objective is concave in ℓ with derivative $\min\{u'(c_2^c+\ell), 1\} - \beta R_E$, which for $\ell \leq x \leq \bar{\ell}_0$ is at least $\min\{u'(c_2^c + \bar{\ell}_0) - \beta R_E,\ 1 - \beta R_E\} \geq 0$, by scarcity and $\beta R_E \leq 1$, strictly if $R_E < R^*$. Hence $\ell = x$ is optimal, uniquely so if $R_E < R^*$; $V(x) = \Phi(c_2^c + x) - \beta R_E x$ is concave and continuously differentiable on $[0, \bar{\ell}_0]$, with $V'(x) = \min\{u'(c_2^c + x), 1\} - \beta R_E$, which at $x = \bar{\ell}(h_O)$ is $\min\{\Delta_\ell(h_O), 1 - \beta R_E\}$; and $b = \max\{0, c^\dagger - c_2^c - x\} > 0$ if and only if $u'(c_2^c + x) > 1$.

(ii) Let $A^b(x) \equiv V(x) - V(0)$. The proof of Lemma 1 applies with $V(\bar{\ell}(h_O))$ in place of $u(c_2^c) + A(\bar{\ell}(h_O))$: the allocation moves the crisis state only through $\bar{\ell}(h_O)$, so $\mathcal{W}^b(h_O) = u(c_0(h_O)) + \beta^2 F \pi A^b(\bar{\ell}(h_O))$ up to a constant. By (i), A^b is concave and, when $R_E < R^*$, strictly increasing, with $A^b(0) = 0$ and $\frac{d}{dh}A^b(\bar{\ell}(h)) = -\lambda d_O \Delta_\ell^b(h)$ for $h < h_{\mathrm{a}}$. The proof of Proposition 3 uses only these properties of the access term: strict concavity of $\mathcal{W}^b$ on the access-preserving branch follows from the term $u''(c_0)(\beta^* F d_O)^2 < 0$ in (A.2) alone; $G^b(h;\pi) \equiv u'(c_0(h))\beta^* - \beta^2\pi\lambda\Delta_\ell^b(h)$ is continuous and strictly decreasing in h, and strictly decreasing in π because $\Delta_\ell^b > 0$, so its root $h_O^\circ(\pi)$ is strictly decreasing in π, which replaces the implicit-function step; the right derivative of $\mathcal{W}^b$ at h_{a} exceeds the left by $\beta^2 F\pi\lambda d_O \Delta_\ell^b(h_{\mathrm{a}}^-) > 0$; and $\mathcal{W}^b(h;\cdot)$ is affine in π with slope $\beta^2 F A^b(\bar{\ell}(h))$, positive for $h < h_{\mathrm{a}}$. The thresholds are those of Proposition 3 with Δ_ℓ^b in place of Δ_ℓ, and $\pi_p^b \geq \pi_p$ because $\Delta_\ell^b \leq \Delta_\ell$. The proof of Corollary 2 uses only that $A^b(\bar{\ell}(\cdot))$ is strictly decreasing on $[\underline{h}, h_{\mathrm{a}})$ and zero beyond, which holds.

(iii) If $R_E = R^*$, then $1 - \beta R_E = 0$ and, by (i), $V' \equiv 0$ on $[0, \bar{\ell}_0]$, so $A^b \equiv 0$: $\mathcal{W}^b(h_O) = u(c_0(h_O))$ up to a constant, strictly increasing by Lemma 1, with maximizer $\overline{h}$ for every π; at date 1, with c_0 sunk, continuation utility is the same at every feasible allocation. □

S.4.4 Proof of Proposition S.3

Since $\mathcal{R} \leq d_P$, feasible allocations are indexed by $h_O \in [0, \overline{h}]$, with $h_P = (\mathcal{R} - h_O d_O)/d_P \in [0,1]$. Let investors anticipate such an allocation, so that $R^* q_P = 1 - F h_P > 0$ and $R^* q_O = 1 + g_O$ (Lemma 2), both predetermined. Given these prices, the gap of the rule at a feasible allocation (h'_P, h'_O),

$$\frac{h'_P - F h_P}{1 - F h_P} - \frac{h'_O + g_O}{1 + g_O},$$

is strictly decreasing in h'_O along (6), so, as in the proof of Proposition 2, the rule returns its zero if there is one, $h'_O = 0$ if it is negative throughout, and $h'_O = \overline{h}$ if it is positive throughout; the anticipated allocation is an equilibrium if and only if $\tilde{\psi}(h_O) = 0$, or $h_O = 0$ and $\tilde{\psi}(0) \leq 0$, or $h_O = \overline{h}$ and $\tilde{\psi}(\overline{h}) \geq 0$, where

$$\tilde{\psi}(h_O) \equiv \frac{(1-F)h_P}{1 - F h_P} - \frac{g_O + h_O}{1 + g_O}, \qquad h_P = \frac{\mathcal{R} - h_O d_O}{d_P},$$

is the gap at the anticipated allocation itself. The first term is strictly increasing in h_P, with derivative $(1-F)/(1-Fh_P)^2$, and h_P is strictly decreasing in h_O, so $\tilde{\psi}$ is continuous and strictly decreasing on $[0, \overline{h}]$. At $h_O = \overline{h}$, either $\overline{h} = \mathcal{R}/d_O$, where $h_P = 0$ and $\tilde{\psi} = -(g_O + \overline{h})/(1 + g_O)$, or $\overline{h} = 1$, where $h_P < 1$ and $\tilde{\psi} = (h_P - 1)/(1 - F h_P)$; in both cases $\tilde{\psi}(\overline{h}) < 0$, so $\overline{h}$ is never an equilibrium. At $h_O = 0$, $h_P = \mathcal{R}/d_P$ and $\tilde{\psi}(0)$ has the sign of $(1+g_O)(1-F)\mathcal{R} - g_O(d_P - F\mathcal{R}) = (1 - F + g_O)\mathcal{R} - g_O d_P$. Hence, if $\mathcal{R} \leq g_O d_P/(1 - F + g_O)$, the unique equilibrium is $h_O = 0$; otherwise $\tilde{\psi}(0) > 0 > \tilde{\psi}(\overline{h})$ and the unique equilibrium is the unique zero of $\tilde{\psi}$ in $(0, \overline{h})$.

Closed form. At the zero, solving $\tilde{\psi} = 0$ for h_O gives $h_O = H(h_P)$, with $H(p) \equiv [(1 - F + g_O)p - g_O]/(1 - Fp)$ the second expression displayed in the proposition, and (6) reads $d_P h_P + d_O H(h_P) = \mathcal{R}$. Multiplying by $1 - F h_P > 0$, h_P is a root in $(0,1)$ of $Q(p) \equiv F d_P p^2 - Bp + C$. Since $Q(0) = C > 0$ and $Q(1) = -(1 - F)(D - \mathcal{R}) < 0$, Q has exactly one root in $(0,1)$, its smaller one, which after rationalization is the first expression displayed.

Comparison. At $\kappa = 1$, Proposition 2(i) gives $h_O = L(h_P) \equiv (1 - F)h_P - g_O$ on its interior range and exemption if and only if $\mathcal{R} \leq g_O d_P/(1-F)$, a cutoff weakly above $g_O d_P/(1 - F + g_O)$. For $p \in (0,1]$,

$$H(p) - L(p) = \frac{(1 - F)\, p\, (g_O + Fp)}{1 - Fp} > 0,$$

so $d_P p + d_O H(p) > d_P p + d_O L(p)$ for $p > 0$, both sides being strictly increasing in p. Where both rules are interior, the rate-normalized rule therefore selects the smaller private haircut and the larger official one; where only it is interior, $g_O d_P/(1 - F + g_O) < \mathcal{R} \leq g_O d_P/(1 - F)$, its official haircut is positive against zero; and where neither is, both are zero. □

S.4.5 Derivation of the Welfare Function (S.4)

The proof of Lemma 1 applies with two changes. On the restructuring branch, date-2 consumption is $c_2^c + \ell_O + \ell_B$ in state y^ℓ and $y^h - \bar{M} + \ell_B$ in state y^h, with both draws repaid at settlement; by Definition S.1, which nets out the repayments, the two states contribute $u(c_2^c) + A^\ell(\bar{\ell}(h_O), \bar{\ell}_B(h_B))$ and $u(y^h - \bar{M}) + A_B^h(\bar{\ell}_B(h_B))$, while the repayment branch, on which both facilities are unimpaired, still contributes a constant. At date 0, $q_P d_P = \beta^*(d_P - F h_P d_P)$ and $h_P d_P = \mathcal{R} - h_O d_O - h_B d_B$ give $c_0(h_O, h_B)$ as stated, with $\partial c_0/\partial h_i = \beta^* F d_i > 0$ for $i \in \{O, B\}$. Discounting the allocation-dependent terms to date 0 yields (S.4). □

S.4.6 Proof of Proposition S.4 and Remark S.3

Work in relief units $r_i \equiv h_i d_i$, on $K \equiv \{r \in \mathbb{R}^3_+ : \ r_P + r_O + r_B = \mathcal{R}, \ r_i \le d_i\}$, and write $\bar{x}_O \equiv \max\{0, \bar{\ell}_0 - \lambda r_O\}$ and $\bar{x}_B \equiv \max\{0, \bar{\ell}_{B0} - \gamma r_B\}$ for the capacities. Because $\mathcal{R} \le d_P$, relief can be moved from either official class to bondholders within K.

Binding caps. Let $(\bar{x}_O, \bar{x}_B) \le (\bar{\ell}_0, \bar{\ell}_{B0})$. Since u' is decreasing, at feasible draws the objective in (S.3) has partial derivatives of at least $u'(c_2^c + \bar{\ell}_0 + \bar{\ell}_{B0}) - \beta \max\{R_E, R_B\}$ and the objective in state y^h a derivative of at least $u'(y^h - \bar{M} + \bar{\ell}_{B0}) - \beta R_B$, both positive by Assumption S.1. So the caps bind, and $\partial A^\ell/\partial \bar{x}_O = \Delta_\ell$, $\partial A^\ell/\partial \bar{x}_B = \Delta_B^\ell$ and $(A_B^h)' = \Delta_B^h$ are strictly positive.

Marginal costs and concavity. In relief units, (S.4) has $c_0 = y_0 + \beta^*(d_P - F r_P) + d_O/R_O + d_B/R_B$, and $\mathcal{W}$ is continuous in r. Let $\gamma > 0$. On the convex set $\bar{K} \equiv \{r \in K : \ \lambda r_O \le \bar{\ell}_0, \ \gamma r_B \le \bar{\ell}_{B0}\}$ the capacities are affine in r, and $\mathcal{W}$ is an affine function plus positive multiples of $u(c_0)$, $u(c_2^c + \bar{x}_O + \bar{x}_B)$ and $u(y^h - \bar{M} + \bar{x}_B)$, whose arguments have the linearly independent gradients $-\beta^* F(1,0,0)$, $-(0,\lambda,\gamma)$ and $-(0,0,\gamma)$; as at least one of them varies along any segment, $\mathcal{W}$ is strictly concave on $\bar{K}$. Where both windows retain capacity, $\bar{x}_O, \bar{x}_B > 0$, it is differentiable with $\partial \mathcal{W}/\partial r_i = -\sigma_i$, by the previous step.

Part (a). At the date-1 node c_0 is sunk, and by the derivation of (S.4) continuation utility depends on r only through $\pi A^\ell(\bar{x}_O, \bar{x}_B) + (1-\pi) A_B^h(\bar{x}_B)$, strictly increasing in $\bar{x}_O$ and $\bar{x}_B$ by the first step. Since $\bar{\ell}_0 > 0$, moving any $r_O > 0$ to bondholders raises $\bar{x}_O$ to $\bar{\ell}_0$, so every Markov allocation has $r_O = 0$. If $\gamma > 0$ and $\bar{\ell}_{B0} > 0$, the same holds for r_B: the unique Markov allocation is $r = (\mathcal{R}, 0, 0)$, with the largest h_P on K and hence the lowest $q_P = \beta^*(1 - F h_P)$. With an inert window, $\bar{x}_B = \bar{\ell}_{B0}$ on K, so every $r \in K$ with $r_O = 0$ is a Markov allocation.

Part (b). With an inert window, $\sigma_B = 0$ and $\bar{x}_B = \bar{\ell}_{B0}$ on K, so $\mathcal{W}$ varies with r only through r_P and r_O, strictly decreasing in r_P and nonincreasing in r_O: moving relief to B within K raises $\mathcal{W}$ strictly if it comes from P and weakly if it comes from O. Hence a Ramsey allocation with $r_P > 0$ has $r_B = d_B = \min\{d_B, \mathcal{R}\}$, and from one with $r_P = 0$, moving relief from O to B leads to a Ramsey allocation with $r_B = \min\{d_B, \mathcal{R}\}$ (if relief does move, the emergency facility

is exhausted before and after, or $\mathcal{W}$ would rise). A Ramsey allocation exists, because $\mathcal{W}$ is continuous on the compact set K; so one has $r_B = \min\{d_B, \mathcal{R}\}$, and its pair (r_P, r_O) maximizes $\mathcal{W}$ subject to $r_P + r_O = \mathcal{R}' \equiv \max\{0, \mathcal{R} - d_B\}$. Let $\mathcal{R}' > 0$. On that set $c_0 = y_0 + d_B/R_B + \beta^*(d_P - F\mathcal{R}' + Fr_O) + d_O/R_O$, and $A^\ell(\bar{x}_O, \bar{\ell}_{B0})$ equals $u(c_2^c + \bar{\ell}_{B0} + \bar{x}_O) - \beta R_E \bar{x}_O - u(c_2^c + \bar{\ell}_{B0})$ plus a constant, so $\mathcal{W}$ is, up to a constant, the function (12) of the economy of Section 2 with $\mathcal{R}'$, $y_0 + d_B/R_B$ and $c_2^c + \bar{\ell}_{B0}$ in place of $\mathcal{R}$, y_0 and c_2^c. By the second condition of Assumption S.1, that economy satisfies the scarcity condition of Assumption 4, and the proof of Proposition 3 uses nothing else; its part (ii) applies, since $\mathcal{R}' < d_P$ and $\bar{\ell}_0 > 0$.

Part (c). Let $\gamma > 0$ and $\bar{\ell}_{B0} > 0$, and let r be a Ramsey allocation with $\bar{x}_O, \bar{x}_B > 0$. Since $\mathcal{W}$ is differentiable at r and the constraints defining K are linear, the Kuhn–Tucker conditions are necessary: there are multipliers μ on the relief constraint and $\nu_i^0, \nu_i^1 \geq 0$ on $r_i \geq 0$ and $r_i \leq d_i$, with complementary slackness, such that $\sigma_i = \mu + \nu_i^0 - \nu_i^1$ for each class. This gives the three cases, with $\mu \geq \sigma_i > 0$ for some class with $r_i > 0$. If $r_P \in (0, d_P)$, then $\mu = \sigma_P = \beta^* F u'(c_0)$: an interior r_B requires $\sigma_B = \sigma_P$, which is (S.5) after division by F, and $\beta\gamma\bar{\Delta}_B > u'(c_0)$ means $\sigma_B > \mu$, which rules out $r_B > 0$. The conditions are also sufficient on $\bar{K}$: for $r' \in \bar{K}$, concavity gives $\mathcal{W}(r') - \mathcal{W}(r) \leq -\sum_i \sigma_i(r_i' - r_i) = -\sum_i \nu_i^0 r_i' - \sum_i \nu_i^1 (d_i - r_i') \leq 0$. By strict concavity, at most one allocation with $\bar{x}_O, \bar{x}_B > 0$ satisfies them, and it maximizes $\mathcal{W}$ on $\bar{K}$. If $\lambda \min\{d_O, \mathcal{R}\} < \bar{\ell}_0$ and $\gamma \min\{d_B, \mathcal{R}\} < \bar{\ell}_{B0}$, then $\bar{K} = K$ and both windows retain capacity on all of K, so the conditions characterize the unique Ramsey allocation.

Remark S.3. Fix r_O with $\bar{x}_O > 0$, so that $r_P = \mathcal{R} - r_O - r_B$. On the branch $\bar{x}_B > 0$, and using $\beta = \beta^*$, (S.5) reads $G(r_B; \gamma, R_B, \pi) \equiv u'(c_0) - \beta\gamma\bar{\Delta}_B = 0$. Write $\bar{u}'' \equiv \pi u''(c_2^c + \bar{x}_O + \bar{x}_B) + (1 - \pi)u''(y^h - \bar{M} + \bar{x}_B) < 0$, by Assumption 1. Since $\partial c_0/\partial r_B = \beta^* F$, $\partial \bar{x}_B/\partial r_B = -\gamma$, $\partial \bar{x}_B/\partial \gamma = -r_B$ and $\partial c_0/\partial R_B = -d_B/R_B^2$,

$$G_{r_B} = \beta^* F u''(c_0) + \beta\gamma^2 \bar{u}'' < 0, \qquad G_\gamma = -\beta\bar{\Delta}_B + \beta\gamma r_B \bar{u}'' < 0,$$
$$G_{R_B} = \beta^2\gamma - u''(c_0)\, d_B/R_B^2 > 0, \qquad G_\pi = \beta\gamma\big(\Delta_B^h - \Delta_B^\ell\big).$$

By the implicit function theorem, the derivative of the solution with respect to each parameter has the sign of the corresponding derivative of G: negative for γ; positive for R_B, hence negative for $g_B = R^*/R_B - 1$; and, for π, the sign of $\Delta_B^h - \Delta_B^\ell = u'(y^h - \bar{M} + \bar{x}_B) - u'(y^\ell - \bar{M} + \bar{x}_O + \bar{x}_B)$, which is positive if and only if $\bar{x}_O > y^h - y^\ell$.

Discussion. At the allocation of part (b), $h_B = 1$ if $\mathcal{R} \geq d_B$, and $h_P = h_O = 0 < h_B$ otherwise. If $\mathcal{R} > d_B$, the relief constraint gives $h_O \leq h_P$ iff $h_O \leq (\mathcal{R} - d_B)/(d_P + d_O)$, the equal-treatment haircut of the residual problem, which lies above h_O under preferred creditor status and below it under full breach. In the inert case a Markov equilibrium with bilateral haircut h_B has $h_P = (\mathcal{R} - h_B d_B)/d_P$, against $\mathcal{R}/d_P$ under conditioning, and (2) gives the price

difference. Finally, $\beta R_B = 1 - \mathrm{GE}_B$, so Δ_B^ℓ and Δ_B^h each equal $(u'-1) + \mathrm{GE}_B$, with u' evaluated at the consumption of the state. □

S.4.7 Proof of Corollary S.2

As in Lemma 2, $\chi_B = 1 - R^*/R_B = -g_B$, and $\chi_P = Fh_P$ at an anticipated profile.

The rule. Given the compensations, the relief $\sum_i d_i \min\{1, \max\{0, n + \kappa\chi_i\}\}$ is continuous and nondecreasing in n, from 0 to $D > \mathcal{R}$: a level exists, and levels with the same relief return the same profile, every term being nondecreasing. A class with $h_i = 0$ has netted loss $-\kappa\chi_i \geq n$, and a class with $h_i = 1$ has $1 - \kappa\chi_i \leq n$. With two classes, the netted gap of Definition 4, $(h_P - \kappa\chi_P) - (h_O - \kappa\chi_O)$, is therefore zero at the profile if no bound binds. Otherwise a bound binds, and $0 < \mathcal{R} < D$ excludes $h_P = h_O \in \{0, 1\}$: if $h_O = 0$ or $h_P = 1$, then $h_O = \underline{h}$ and the gap is nonpositive; if $h_O = 1$ or $h_P = 0$, then $h_O = \overline{h}$ and it is nonnegative. As the gap is strictly decreasing in h_O along (6), the profile minimizes its absolute value.

Equilibrium. A feasible profile is an equilibrium if and only if

$$h_i = \max\{0,\ (1 - \kappa F)h_P - \kappa g_i\}, \qquad i \in \{O, B\}. \tag{S.7}$$

Feasibility and $\mathcal{R} \leq d_P$ give $h_P \leq \mathcal{R}/d_P \leq 1$. If (S.7) holds, then $h_P > 0$, as $h_P = 0$ would give no relief, and the level $n = (1 - \kappa F)h_P$ returns the profile. Conversely, let the rule return the anticipated profile, with level n. Then $h_P > 0$, as $h_P = 0$ requires $n \leq 0$ and hence $h_O = h_B = 0$. Either $h_P = n + \kappa F h_P$, so that $n = (1 - \kappa F)h_P \leq 1$ and the rule gives $h_i = \max\{0, n - \kappa g_i\}$, which is (S.7); or $h_P = 1 < n + \kappa F$, in which case $\mathcal{R} = d_P$ and $h_O = h_B = 0$, so $n \leq \kappa g_i$ and $(1 - \kappa F) - \kappa g_i < 0$, which is (S.7) again. Under (S.7) the relief constraint reads $\Phi(h_P) = \mathcal{R}$, where $\Phi(h) \equiv h d_P + \sum_{i \in \{O,B\}} \max\{0, (1 - \kappa F)h - \kappa g_i\} d_i$ is continuous and strictly increasing with $\Phi(0) = 0 < \mathcal{R} \leq \Phi(\mathcal{R}/d_P)$. Its root lies in $(0, \mathcal{R}/d_P]$, so (S.7) gives $h_i \in [0, 1]$ and the profile is feasible. Hence there is exactly one equilibrium. For $h > 0$, each term $\max\{0, (1 - \kappa F)h - \kappa g_i\}$ is nonincreasing in κ, and strictly decreasing where it is positive. Hence the equilibrium h_P is nondecreasing in κ, and strictly increasing while an official class shares in relief; on that range the issue price $q_P = \beta^*(1 - Fh_P)$ is strictly decreasing in κ.

Threshold. At an anticipated profile the three netted losses are $(1 - \kappa F)h_P$, $h_O + \kappa g_O$ and $h_B + \kappa g_B$. They are equal iff $h_i = (1 - \kappa F)h_P - \kappa g_i$ for $i \in \{O, B\}$, in which case the relief constraint reads $h_P\big[d_P + (1 - \kappa F)(d_O + d_B)\big] = \mathcal{R} + \kappa(g_O d_O + g_B d_B)$, with a strictly positive coefficient, so (S.6) is the unique solution. The numerator of h_P^{nw} is positive and nondecreasing in κ and its denominator positive and strictly decreasing, as $F > 0$; hence h_P^{nw} is strictly increasing from $h_P^{\mathrm{nw}}(0) = \mathcal{R}/D = \theta$. Since $h_O^{\mathrm{nw}} = h_B^{\mathrm{nw}} + \kappa(g_B - g_O)$, the relief constraint reads $(d_O + d_B)\, h_B^{\mathrm{nw}} = \mathcal{R} - d_P h_P^{\mathrm{nw}} - \kappa(g_B - g_O)d_O$, so h_B^{nw} is continuous and, as $g_B \geq g_O$, strictly

decreasing from $h_B^{\text{nw}}(0) = \theta > 0$. This delivers κ_3^*, equal to $\min\{1, \kappa^*\}$ if $g_B = g_O$: then $h_B^{\text{nw}} = h_O^{\text{nw}}$ has the sign of $(1-\kappa F)\mathcal{R} - \kappa g_O d_P$.

Parts (i) and (ii). Let $\kappa \leq \kappa_3^*$. Then $h_O^{\text{nw}} \geq h_B^{\text{nw}} \geq 0$, so (S.6) satisfies (S.7) and $\Phi(h_P^{\text{nw}}) = \mathcal{R}$: it is the equilibrium. Moreover, $h_P^{\text{nw}} - h_i^{\text{nw}} = \kappa(F h_P^{\text{nw}} + g_i) > 0$ for $\kappa > 0$, $h_O^{\text{nw}} - h_B^{\text{nw}} = \kappa(g_B - g_O)$, and (S.6) gives $h_P^{\text{nw}} = h_O^{\text{nw}} = h_B^{\text{nw}} = \theta$ at $\kappa = 0$. Finally, $(1-\kappa F)\mathcal{R}/d_P \geq (1-\kappa F)h_P^{\text{nw}} \geq \kappa g_O$, that is, $\kappa \leq \kappa^*$; hence $\kappa_3^* \leq \kappa^*$. Now let $\kappa > \kappa_3^*$. The solution (S.6) has $h_B^{\text{nw}} < 0$, so no feasible profile equalizes the three netted losses, and the equilibrium has $h_B = 0$: otherwise $h_O \geq h_B > 0$ by (S.7), so the profile solves $h_i = (1-\kappa F)h_P - \kappa g_i$ and the relief constraint, whose only solution (S.6) has $h_B^{\text{nw}} < 0$. With $h_B = 0$, either $h_O = (1-\kappa F)h_P - \kappa g_O \geq 0$ and $h_P d_P + h_O d_O = \mathcal{R}$, a linear system solved by the pair stated; or $h_O = 0$ and $h_P = \mathcal{R}/d_P$. The first requires $(1-\kappa F)\mathcal{R} \geq \kappa g_O d_P$ and the second the reverse inequality, that is, $\kappa \leq \kappa^*$ and $\kappa \geq \kappa^*$ respectively. □

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